Exercise 85

For the following exercises, for each polynomial, a. find the degree; b. find the zeros, if any; c. find the y-intercept(s), if any; d. use the leading coefficient to determine the graph's end behavior; and e. determine algebraically whether the polynomial is even, odd, or neither.

$$f(x) = \frac{1}{2}x^2 - 1$$

Solution

Part (a)

The degree of the polynomial is 2 because the highest power of x is 2.

Part (b)

Set $f(x) = 0$.

$$f(x) = \frac{1}{2}x^2 - 1 = 0$$

Solve for x.

$$\frac{1}{2}x^2 = 1$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

Therefore, the zeros are

$$x = \{-\sqrt{2}, \sqrt{2}\}.$$

Part (c)

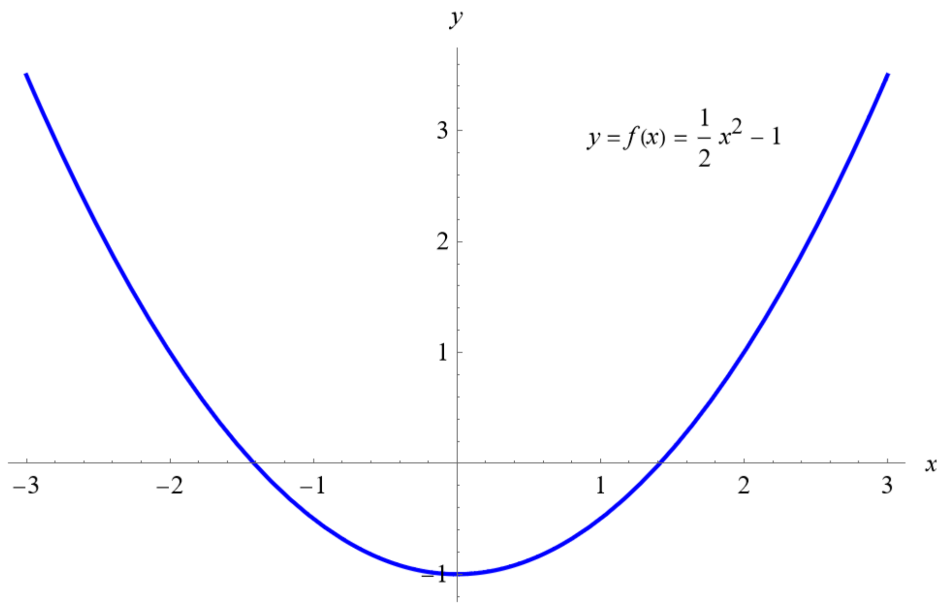
y-intercepts are the points where the function touches the y-axis, which occurs when $x = 0$.

$$f(0) = \frac{1}{2}(0)^2 - 1 = -1$$

Therefore, there's one y-intercept: $(0, -1)$.

Part (d)

$(1/2)x^2$ is the dominant term in the polynomial, so the graph is in the shape of a parabola. Since the coefficient is $+1/2$, it opens upward towards the positive y-axis. The graph of $f(x)$ versus x below illustrates this.



Part (e)

Plug in $-x$ for x in the function.

$$\begin{aligned} f(-x) &= \frac{1}{2}(-x)^2 - 1 \\ &= \frac{1}{2}x^2 - 1 \end{aligned}$$

Since $f(-x) = f(x)$, the function $f(x)$ is even.

Since $f(-x) \neq -f(x)$, the function $f(x)$ is not odd.